Statistical Model Development Of Crater Thermal Inertia From TES

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8^{th} **Planetary Crater Consortium**

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Presentation Outline

Introduction

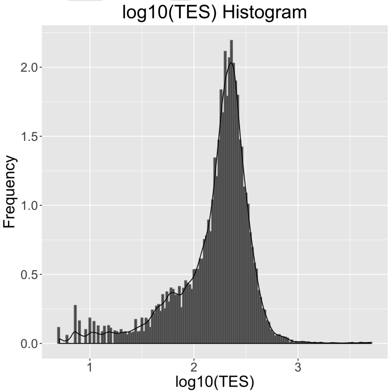
- Thermal inertia (TI) of the Martian surface
  - Derived from brightness temperature measurements
  - Data from Mars Global Surveyor Thermal Emission Spectrometer (TES)
- Purpose to study trends in thermal inertia of impact craters
  - Use statistical model
  - Test if trends are related to crater location, size, and morphological variables
- Sufficient TI statistifcal model must
  - Accurately describe the TI probability distribution function
  - Identify the roles of crater morphology variables
  - Be stable across all combinations of morphology variables

Thermal Inertia Probability Distribution Functions

- TI probability distribution shows at least 2 modes
- Bimodel mixture PDF

$$f_Y(y) = \phi f_{Y_1}(y_1) + (1-\phi) f_{Y_2}(y_2),$$

- $f_{Y_1}(y_1)$ = Gaussian
- $f_{Y_2}(y_2)$ = gamma
- ϕ is the proportion of $f_{Y_1}(y_1)$ and $f_{Y_2}(y_2)$



Thermal Inertia Probability Distribution Function

- Trial models of from 1 to 4 components were tested
- 2-component Gaussian and gamma PDF best fit TI model

$$f_Y(y) = \phi f_{Y_1}(y_1|\mu,\sigma^2) + (1-\phi) f_{Y_2}(y_2|\theta,\kappa)$$
$$= \phi \left[\frac{1}{\sqrt{2\pi\sigma^2}} \exp\left(-\frac{(y_1-\mu)^2}{2\sigma^2} \right) \right] + (1-\phi) \left[\frac{1}{\theta^\kappa \Gamma(\kappa)} y_2^{\kappa-1} \exp(-y_2/\theta) \right],$$

where $-\infty < y_1 < +\infty,\ -\infty < \mu < +\infty,\ \sigma^2 > 0,$
$y_2 > 0, \theta > 0, \kappa > 0.$

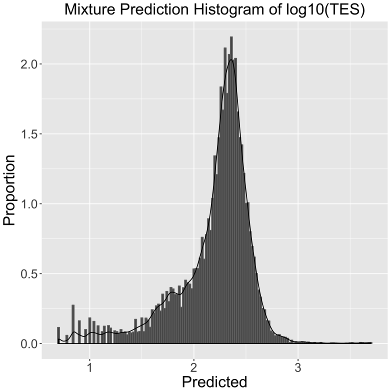
- As observed, $y_1 > 0$, and $\mu > 0$
- The parameters $\mu,\ \sigma^2,\ \theta,\ \kappa$, and ϕ all are estimated using the Expected-Maximization (E-M) method.

Model Outcome Probability Distribution Function

- 2 TI modes as in raw data
- Bimodel mixture PDF

$$\hat{f}_Y(y) = \hat{\phi}\hat{f}_{Y_1}(y_1) + (1-\hat{\phi})\hat{f}_{Y_2}(y_2),$$

- $\hat{f}_{Y_1}(y_1)$ = Gaussian
- $\hat{f}_{Y_2}(y_2)$ = gamma
- $\hat{\phi}$ estimated proportion of $f_{Y_1}(y_1)$ and $f_{Y_2}(y_2)$



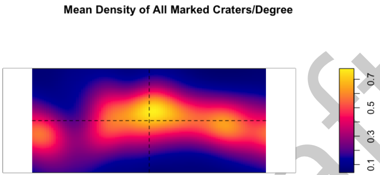
Spatial Density

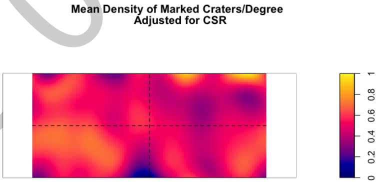
- Mars global region
  0° to 360° longitude by $+80^\circ$ to -80° latitude
- Generated spatially random lat/lon and diameters ($\sim$Mars PF)
- Crater density a ratio of actual to random distributions

$$p(x) = \rho \frac{I_{map}(x)}{I_{map}(x) + I_{rand}(x)},$$

  $p(x)$ the probability of nonrandom crater spatial density,
  τ scales $0 \leqslant p(x) \leqslant 1$,
  $I_{map}(x)$ is the mapped crater spatial density,
  $I_{rand}(x)$ is the spatially random density
- The location x from a lattice overlaying Mars global region
- $p(x)$ value near 0.5 indicates spatial randomness
- Probability near 0 suggests a spatially sparse area
- Probability near 1 indicates spatially abundant area

Spatial Density





TI Model

- Purpose of modeling TI of craters
  - Identify the significant crater morphology variables
  - Predict TI for individual craters from morphology variables
  - Individual craters are classified by variable sizes and levels
- TI model (so far):

$$\begin{aligned} \mathrm{TI} &= \mathrm{TIME} + \mathrm{LAYERS} + \mathrm{TERMINATE} + \mathrm{EDGE} \\ &+ \mathrm{TESTYPE} + \mathrm{p} + \mathrm{DIAMETER} + \mathrm{p} \times \mathrm{DIAMETER}. \end{aligned}$$

- Response variable is the thermal inertia (TI) for each crater

TI Model Explanatory Variables

- Measured explanatory morphology variables are
  - Crater spatial density, p
  - Crater diameter
  - TESTYPE (maximum, median, and minimum)
- Morphological grouping variables for each crater are
  - LAYERS (single, double, or multiple)
  - Layer TERMINATE (pancake and rampart)
  - Layer EDGE (broad lobe and small lobe).

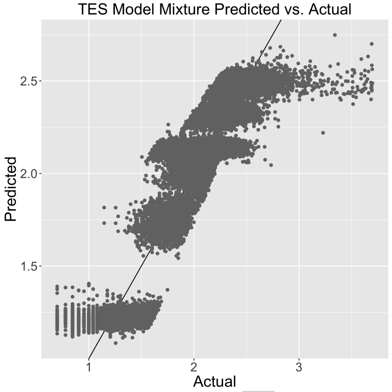
TI Model Explanatory Variables

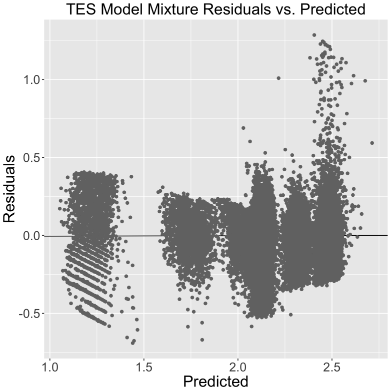
- "Mixture": Gaussian and gamma mixture parameters
- Variables have same effect size
- "Model.TI"
  - Unit step changes for categorical variables
  - 25% change in crater density
  - 10km change in crater diameter
  - Interaction combined 25% density change and 10km diameter change
- Closeness suggests further investigation

Table: Thermal inertia model anti-log2 the component mixture.

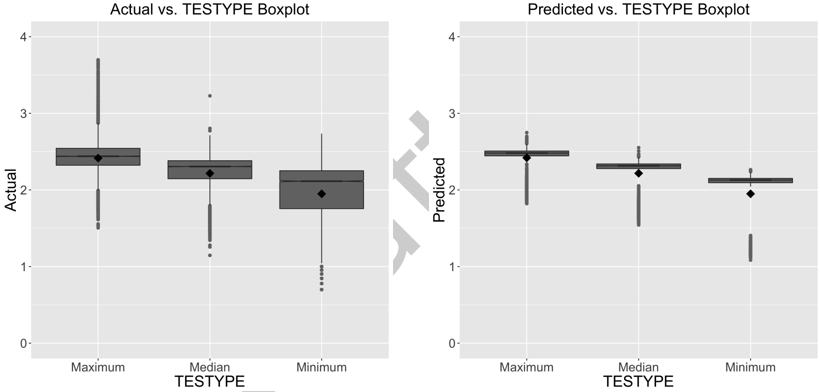
Variables	Mixture	Model.TI
(Intercept)	2.0659	116.3803
TIMENight	1.0085	10.1966
LAYERSMLE	1.0067	10.1551
LAYERSSLE	1.0000	10.0002
TERMINATER	0.9961	9.9096
EDGESL	1.0047	10.1083
TESTYPEMedian	0.9755	9.4512
TESTYPEMinimum	0.9152	8.2260
p	0.2396	1.7362
DIAMETER	0.9996	9.9916
p * DIAMETER	0.2500	1.7785

TI Model Adequacy

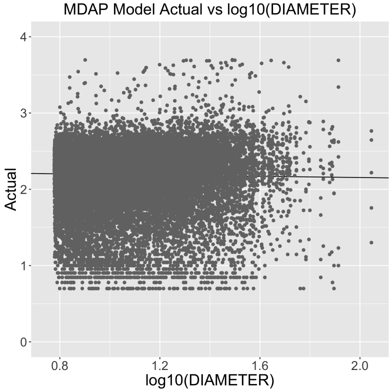


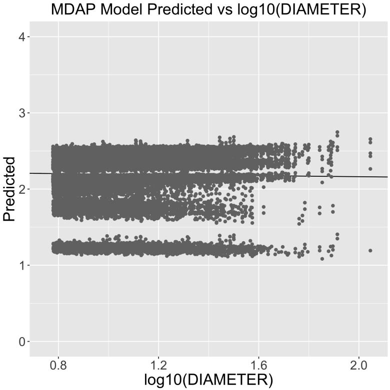


TI Model Adequacy



TI Model Adequacy





Ongoing Work

- Statistical model
    - Describes the crater thermal inertia behavior
    - Craters classified by morphological variables
    - Two-component mixture probability distribution
    - Other PDF combinations need to be examined

Ongoing Work

- The current model adequately reproduces TI values
- Areas for improvement are:
  - Randomization scheme for crater spatial density variable
  - Choice of morphological variables
  - Additional morphological variables should be considered
  - Proxy variables may be required
- Statistical methodology employed demonstrates that individual crater thermal inertia behavior is predictable using the crater's morphological variables as classifiers of thermal inertia commonalities

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