A Generalized Linear Mixed Model for Enumerated Sunspots

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Applied Statistics and Research Methods
Deep Space Exploration Society

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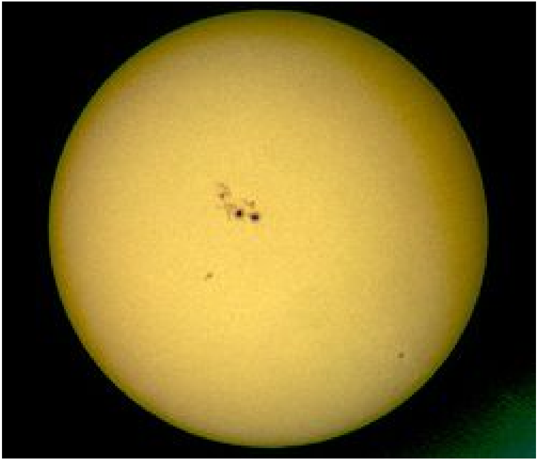
UNIVERSITY of NORTHERN COLORADO

Solar Beauty Spots



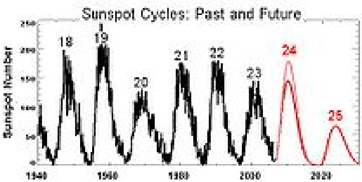
Presentation Outline

- Introduction
- Background
- Wald Approach
- Statistical Models for Counts Data
- Future Development
- Acknowledgments
  Rodney Howe, Solar Bulletin Editor, AAVSO
  Trent Lalonde, Applied Statistics, University of Northern Colorado

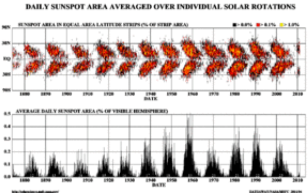
The Physics

- Sunspot generation a current research area
- Sunspots thought to be the visible counterparts of magnetic flux tubes in the Sun's convective zone
- Differential rotation (coriolis effect) stresses the tubes which puncture the Sun's surface
- Energy flux from the Sun's interior decreases and with it surface temperature
- Sunspot activity cycles about every eleven years
- Early in the cycle, sunspots appear in the higher latitudes and then move towards the equator as the cycle approaches maximum: this is called Spörer's law

Sunspot Cycle and Butterfly Plot



(a) Eleven-year cycle



(b) Spörer's Law

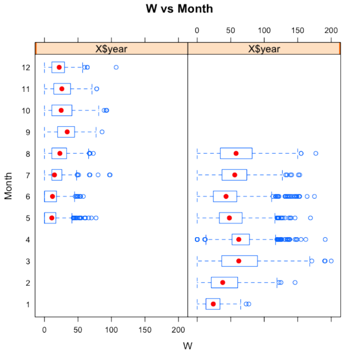
The Astronomy

- First noted sunspots in 364 BC by Chinese astronomer Gan De
- First telescopy in 1610 by English astronomer Thomas Harriot
- Rudolf Wolf established the Wolf Number in 1848
- AAVSO began the American Relative number in 1949
- Overall, weighted monthly count averages are assumed to be unbiased estimates of the true monthly sunspot numbers
- As sunspot cycle in the last 5 months is increasing from a minimum, monthly corrections are anticipated

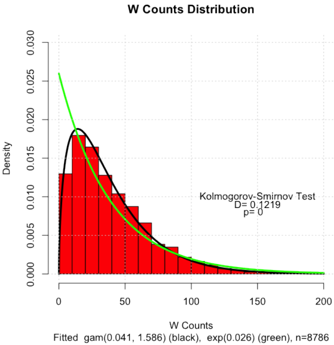
The Statistics

- Multiple observers ($\sim$ 80) worldwide
- Three random variables: sunspot counts, observers, and monthly sunspot numbers
- Sunspot numbers are known to follow an approximately 11-year sinusoidal cycle
- The statistical model needs to tie the average monthly sunspot numbers to the observer-reported counts
- The statistical model should predict sunspot numbers

Monthly Submissions and Histogram



(c) Monthly counts



(d) Histogram with fitted pdfs

Wolf, Wald, and Shapley

The Framers

- Wolf, R, 1848.
  - Developed the Wolf number (a International sunspot number, relative sunspot number, or Zürich number)
  - A quantity measuring the number of sunspots and groups of sunspots on the Sun's surface
  - The relative sunspot number R is computed as

$$R = k(10g + s)$$

  where

    - s is the number of individual spots
    - g is the number of sunspot groups
    - k is a factor that varies with location and instrumentation

The Framers

- Wald, A., The Fitting of Straight Lines if Both Variables are Subject to Error, *Annals Mathematical Statistics*, 1940, Vol. 11, No. 3, pp. 284-300.
  - Response, Y, and predictor, X are random variables
  - Method of least squares (SLR) usually used
  - Fit parameters different for $Y \sim f(X)$ and $X \sim f(Y)$

The Framers

- Shapley, A.H., Reduction of Sunspot-Number Observations, *Publication of the Astronomical Society of the Pacific*, 1949, Vol. 61, No. 358, pp 13-21.
  - Adapted Wald's method to correct observations from many observers to the American Relative sunspot number
  - Correction factor accounts for variations in equipment and seeing conditions
  - A "statistical weight" per observer is also used

The American Relative Sunspot Number

Shapley via Wald

$$R_i = k_i(10g_i + s_i) \tag{1}$$

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$$R_{sm} = \frac{1}{24}\left(R_{a,i-6} + R_{a,i+6} + 2\sum_{j=i-5}^{5} R_{a,j}\right) \tag{3}$$

Poisson Models

Poisson Distribution

Poisson probability distribution function

$$f(y_i; \mu_i) = \frac{e^{-\mu_i} \mu_i^{y_i}}{y_i!} = e^{-\mu_i} \frac{1}{y_i!} e^{y_i \log(\mu_i)}, \quad i = 1, 2, \ldots, N \qquad (4)$$

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GLM canonical link to a monotone function of μ_i

$$\log(\mu_i) = \sum_{i,j} \beta_i x_{ij}, \quad i = 1, \ldots, N, \; j = 1, 2, \ldots, n_i \tag{5}$$

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The matrix form including observer, period, and seeing conditions

$$\log(\boldsymbol{\mu}_{\mathbf{f}}) = \mathbf{X}\boldsymbol{\beta}, \qquad (6)$$

Estimation of R_a

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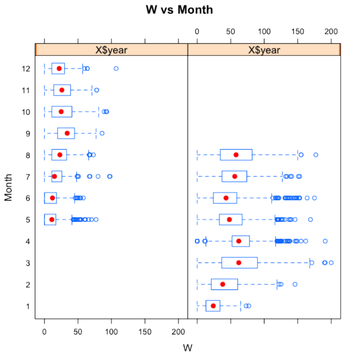
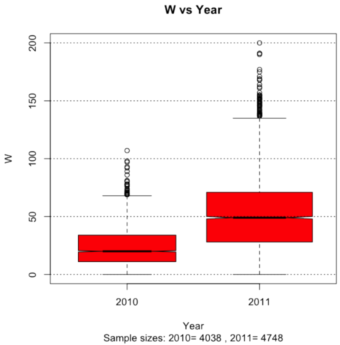
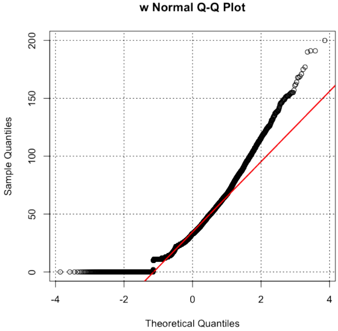


(e) Boxplots of Wolf numbers by Month

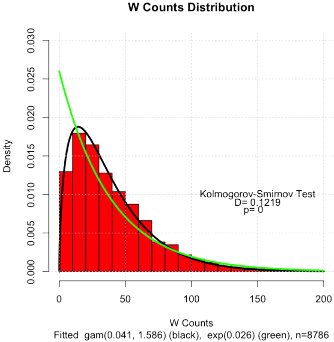


(f) Boxplots of Wolf numbers by Year

Estimation of R_a



(g) The normal Q-Q plot for the Wolf number.



(h) Wolf number distribution.

GLM with Poisson Error Structure

- Several models were fitted using the independent variables observer, seeing conditions, and time sequence
- Two error structures were used: Poisson and negative binomial

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- Several models were fitted using the independent variables observer, seeing conditions, and time sequence
- Two error structures were used: Poisson and negative binomial
- The final model is

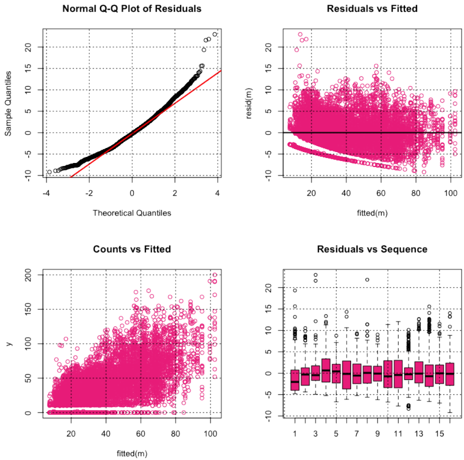
$$\log(\mu_{ij}) = \beta_0 + \beta_1 x_{1ij} + \beta_2 x_{1ij} + \boldsymbol{\beta_3 x_{3ij}} + \eta_{ij},$$

$$x_{1ij} = j^{th} \text{ appearance of the } i^{th} \text{ observer}$$

$$x_{2ij} = j^{th} \text{ occurrence of the } i^{th}\text{observer's seeing condition}$$

$$\mathbf{x_{3ij}} = j^{th} \text{ occurrence of the } i^{th}\text{time sequence}$$

GLM with Poisson Error Structure Diagnostics



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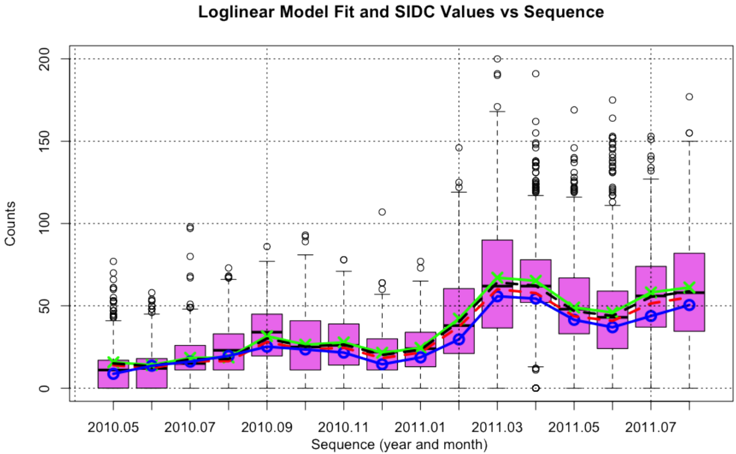
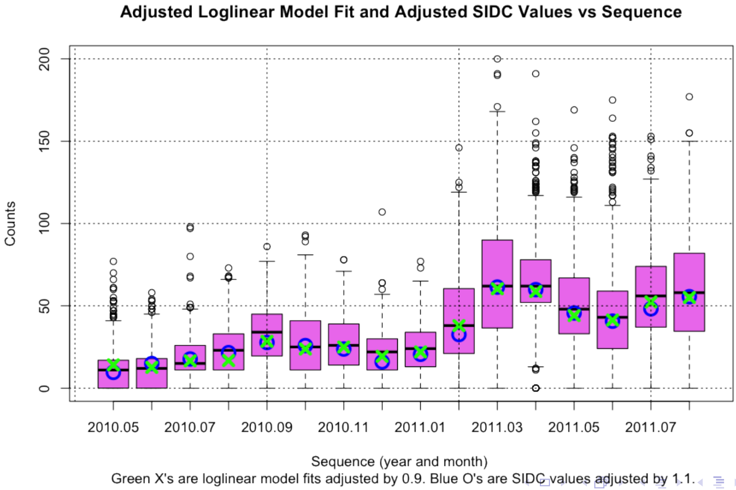


Green X's connected by the curve is the loglinear (LL) model fit. Solid blue curve is SIDC values.
The dashed red curve is a 99% lower CI for LL. The dashed black curve is a 99% upper CI for SIDC.

GLM with Poisson Error Structure Diagnostics

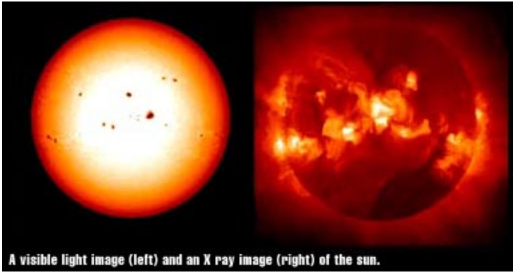


Future Development

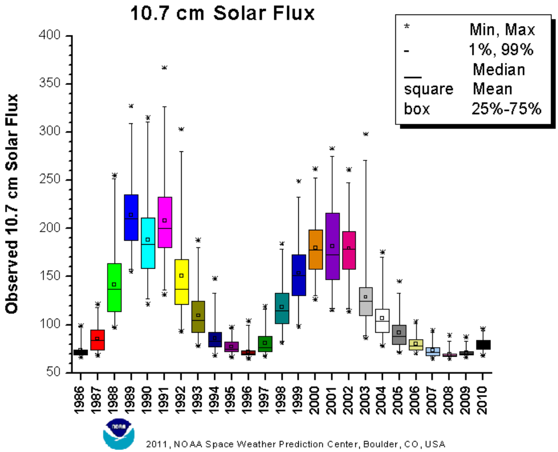
Future Development

- Introduce an observer's equipment factor (fixed)
- Test for the effect of the Solar hemisphere
- Braid in
  - Optical observations from Europe
  - X-ray observations from GOES-15 satellite
  - 10.7cm radio from Deep Space Exploration Society, Canada, and Australia

Soft X-rays



10.7 cm (2800 MHz) Radio



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